Novel coronavirus 2019-nCov: early estimation of epidemiological parameters and epidemic predictions
Supplementary

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1 Within China model

Let y_{it} be the number of new case detections of 2019-nCoV in Chinese city $i = 1, \ldots, n$ modelled as

$$y_{is} \sim \text{Poisson}(\phi_i x_{is})$$

where x_{it} is the expected number of new case detections in city i on day s. $0 \leq \phi_i \leq 1$ is a city-specific case ascertainment probability. In the case of China, we assume that

$$\phi_i = \begin{cases} \phi & \text{if } i = \text{`Wuhan'} \\ 1 & \text{otherwise} \end{cases}$$

Within China, we model an individual's trajectory through infection as susceptible, exposed (and not infectious), infected (and infectious), and finally removed. In continuous time, let S_{it}, E_{it}, I_{it}, and R_{it} be the numbers of susceptible, exposed, infected, and removed individuals respectively in city i at time t. We assume that

$$x_{is} = \int_{(s^+-1)}^{s+} R_{it} \mathrm{d}t$$

where $s+$ denotes the right limit of day s. That is, the number of newly recovered individuals occurring on day s.

We model S_{it}, E_{it}, I_{it}, and R_{it} using a set of ordinary differential equations

$$\begin{aligned} \frac{\mathrm{d}\vec{S}_t}{\mathrm{d}t} &= -\vec{S}_t \odot \vec{\lambda}_t \\ \frac{\mathrm{d}\vec{E}_t}{\mathrm{d}t} &= \vec{S}_t \odot \vec{\lambda}_t - \alpha \vec{E}_t \\ \frac{\mathrm{d}\vec{I}_t}{\mathrm{d}t} &= \alpha \vec{E}_t - \gamma \vec{I}_t \\ \frac{\mathrm{d}\vec{R}_t}{\mathrm{d}t} &= \gamma \vec{I}_t \end{aligned}$$

where $\vec{S}_t = (S_{1t}, \ldots, S_{mt})^T$ and likewise for $\vec{E}_t$, $\vec{I}_t$, and $\vec{R}_t$. Furthermore,

$$\vec{\lambda}_t = \frac{K \cdot \vec{I}_t}{\vec{N}^2}$$

the force of infection vector for each city in China. K is a $n \times n$ matrix where the $k_{ij},\ i \neq j$ element is the mean daily number of passengers flying between city i and j, with diagonal elements $k_{ii} = 1,\ i = 1, \ldots, m$. $\vec{N}$ is a vector of length n containing the population size of each city in China.

2 The rest of the world

Let the number of imported cases of 2019-nCoV in m countries other than China be $z_j s$ for $j = 1, \ldots, m$ where

$$z_{jt} \sim \text{Poisson}(\nu_{jt})$$

and

$$\nu_{jt} = \frac{W_{j\cdot}\vec{I}_t}{\vec{N}}$$

where W is a $m \times n$ matrix with w_{ji}th element being the mean daily number of passengers flying from Chinese city i to non-Chinese city j.

3 Inference

We perform parameter inference on $\vec{\theta} = (\beta, \phi, \gamma, I_{Wuhan,0})^T$ using Maximum Likelihood Estimation, with log likelihood function

$$\ell\left(\vec{y}, \vec{z}; \vec{\theta}\right) \propto \sum_{t=1}^{T} \left[\sum_{i=1}^{n} \{y_{it} \log(\phi_i \lambda_{it}) - \phi_i \lambda_{it}\} + \sum_{j=1}^{m} \{z_{jt} \log \nu_{jt} - \nu_{jt}\} \right].$$

In practice, since the support of our parameters is bounded ($\beta, \gamma, I_{Wuhan,0} > 0,\ \phi > 0$), inference is made on the log scale using the standard Nelder-Mead optimisation routine implemented in R v3.6.1. 95% confidence intervals are calculated assuming asymptotic Normality on the log-scale and transformed onto the linear scale for interpretation purposes.